Inductor and Transformer Design

1 Introduction

The conditioning of power flow in Power Electronic Systems (PES) is done through the use of electromagnetic elements (inductors and transformers). In this section the basics of electromagnetics is reviewed. They are formulated in such a way as to be useful for the design of inductors and transformers.

2 Electromagnetics

The voltage across and the current through a conducting element is related through Ohm's lam. This law may be stated as follows. When an electric field (of intensity ϵ V/m) is set up across a conducting material (of conductivity σ 1/Ω-m), there is an average flow of electrical charges across the conducting material (of current density J A/m). This is shown in Fig. 1.

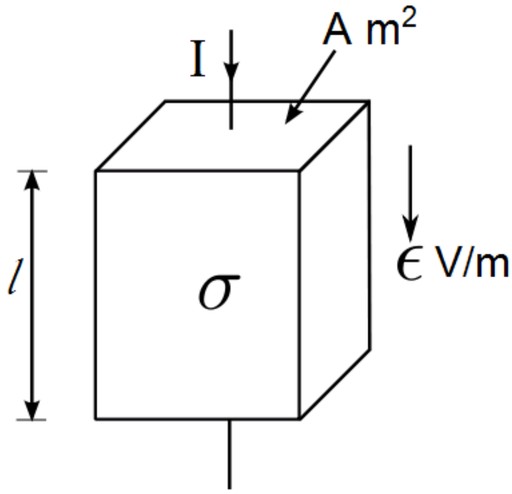


Figure 1

$$J = \sigma\epsilon$$

When expressed in terms of element voltage and current, this reduces to the familiar ststement of Ohm's law.

$$I = V/R \; ; \; R = l/\sigma A$$

In comparison with conducting material, the property of magnetic material may be stated as follows.

When a magnetic field (of intensity H A/m) is set up, across a magnetic material (of permeability μ H/m) a magnetic flux (of density B T/m^2) is established in the magnetic material, as shown in Fig. 2.

$$B = \mu H$$

The above equation, in terms of magnetomotive force (mmf = F) and the flux (Φ) in the magnetic circuit, reduces to

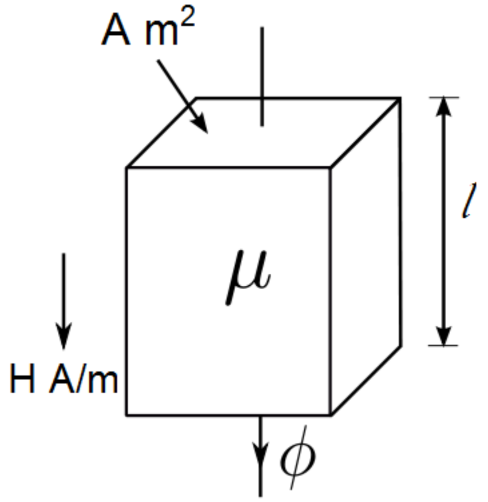


Figure 2

$$\Phi = F/R \text{ ; } R = \text{Reluctance of the magnetic circuit } l/A\mu$$

The above relationship is analogous to Ohm's law for magnetic circuits. The magnetic permebility μ of any magnetic material is usually expressed relative to the permeability of free space ($\mu_o = 4\pi \times 10^{-7}$ H/m).

$$R = l/A\mu_o\mu_r$$

Electromagnetic circuit elements consists of an electric circuit and a magnetic circuit coupled to each other . The electric current in the electric circuit sets up the magnetic field in the magnetic circuit with resultant magnetic flux. Seen as an electric circuit element, the electromagnetic element possesses the property of energy storage without dissipation.

Ampere's law and Faraday's law relate the electric and magnetic circuits of the electromagnetic element. Ampere's law states that the mmf in a magnetic circuit is equal to the electric current enclosed by the magnetic circuit. For example, for the electromagnetic circuits shown in Figs 3 and 4, the magnetic circuit mmf's are I and NI respectively. With further assumption that the magnetic material is isotropic and homogenous and that the magnetic flux distribution is uniform.

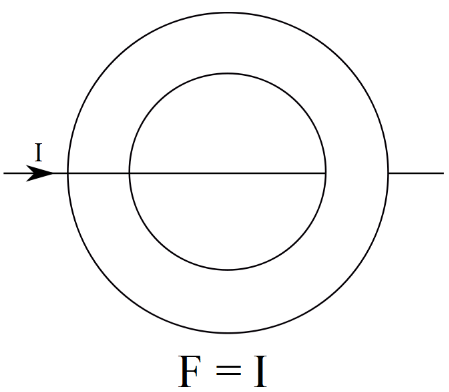


Figure 3

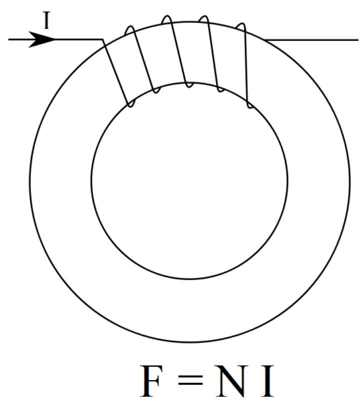


Figure 4

Using Ampere's law, we may relate the magnetic flux in the magnetic circuit as

$$\Phi = (\Sigma \text{ I})/R = NI/R$$

The above equation may conveniently be put in the equivalent circuit shown in Fig. 5. Faraday's law relates the voltage induced in an electric circuit that is coupled to a magnetic circuit.

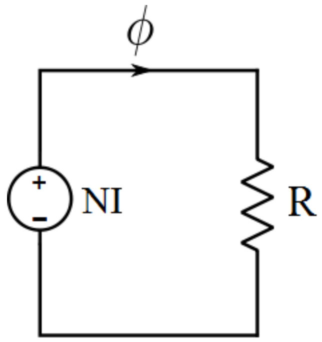


Figure 5

$$v = N\, d\Phi/dt = (N^2/R)\, d\Psi/dt$$

The quantity N^2/R is defined as the inductance of the electric circuit.

$$v = L\, di/dt$$

Thus an electromagnetic circuit provides us an electric circuit element (inductor). The voltage across an inductor is directly proportional to the rate of rise of current through it. The energy stored in the magnetic circuit is

$$Li^2/2 = F^2/2R = \Phi^2 R/2 = \phi F/2$$

The equivalent circuit of an inductor showing both its electric and magnetic parts may be conveniently represented as shown in Fig. 6.

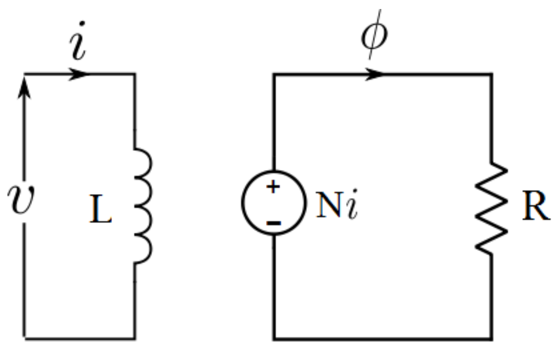


Figure 6

However in practice, the inductor will have certain parasitic resistance (of the wire in the electric circuit) and magnetic leakage (in the magnetic circuit). These non-idealities may be conveniently be incorporated in the equivalent circuit shown in Fig. 7.

The design of an inductor involves the design of the electrical (No. of turns and the wire size) and the magnetic (geometry of the magnetic core and its required magnetic property) circuit.

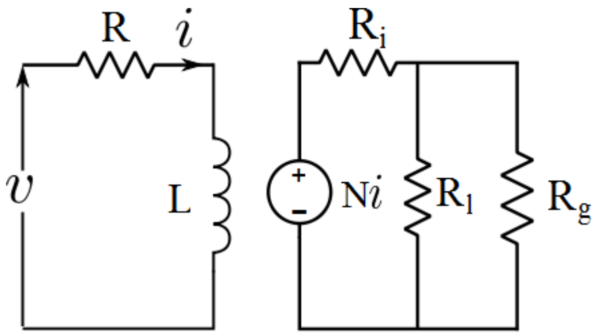


Figure 7

3 Design of Inductor

The inductor consists of a magnetic circuit and a electrical circuit. The design requires,

1. The size of wire too be used for the electric circuit, to carry the rated current safely.
2. The size and shape of the magnetic core to be used such that
   (a) The peak flux is carried safely by the core without saturation.
   (b) The required size of the conductors are safely accommodated in the core.
3. The number of turns of the electric circuit to obtain the desired inductance.

4 Material constraints

Any given wire (conducting material) can only carry a certain maximum current per unit cross section of the wire size. When this limit is exceeded, the wire will overheat from the heat generated ($\mathrm{I^2R}$) and melt or deteriorate. The safe current density for the conducting material is denoted by J $\mathrm{A/m^2}$.

Any magnetic material can only carry a certain maximum flux density. When this limit is exceeded, the material saturates and the relative permeability μ_{r} drops substantially. This maximum allowable flux density for the magnetic material is denoted by B_m $\mathrm{T/m^2}$.

In order to design an inductor of L Henry, capable of carrying an rms current of $\mathrm{I_{rms}}$ and peak current of $\mathrm{I_p}$,

$$\text{Let the wire size be } \mathrm{a_w}\ \mathrm{m^2} : \mathrm{a_w} = \mathrm{I_{rms}/J}$$
$$\text{Let the peak flux density in the core of area } (\mathrm{A_c}) \text{ be } \mathrm{B_m}\ \mathrm{Wb/m^2}$$

$$\mathrm{LI_p} = \mathrm{N\Phi_p} = \mathrm{N}A_{\mathrm{c}}\mathrm{B_m} \tag{1}$$

The winding is accommodated in the window of the core. Let the window area ($\mathrm{A_w}$) be filled by conductors to a fraction of $\mathrm{k_w}$

$$\mathrm{k_wA_w} = \mathrm{Na_w} = \mathrm{NI_{rms}/J} \tag{2}$$

From (1) and (2)

$$\begin{aligned} \mathrm{LI_pNI_{rms}/J} &= \mathrm{NA_cB_mk_wA_w}\ ; \\ \mathrm{LI_pI_{rms}/J} &= \mathrm{k_wJB_mA_cA_w} \end{aligned}$$

Energy handling capacity is proportional to the area product ($A_c \times A_w$) of the core.

- k_w depends on how well the winding can be accommodated in the window of the core. k_w is usually 0.3 to 0.5.
- B_m is the maximum unsaturated flux density for the core material. B_m iss about 1 Wb/m^2 for iron and 0.2 Wb/m^2 for ferrites.
- J is the maximum allowable current density for the conductor. For copper conductors J is between 2×10^6 A/m^2 to 5×10^6 A/m^2.

5 Design steps

Input : L, I_p, I_{rms}, Wire tables, core tables, J, B_m, k_w

1. Compute $A_cA_w = LI_pI_{rms}/k_wJB_m$
2. Select a core from the core tables with the required A_cA_w.
3. For the selected core, find A_c, A_w.
4. Compute $N = LI_p/B_mA_c$. Select nearest whole number of N^*.
5. Compute $a_w = I_{rms}/J$. Select next higher a_w^* from wire tables.
6. Compute $l_g = \mu_o N^* I_p/B_m$.
7. Check for assumptions :
   (a) $R_i \ll R_g$: $l_i/\mu_r \ll l_g$
   (b) No fringing : $l_g \ll \sqrt{A_c}$
8. Recalculate $J^* = I_{rms}/a_w^*$.
9. Recalculate $K_w^* = A_w/N^*a_w^*$.
10. Compute from the geometry of the core, mean length per turn and the length of winding. From wire tables find the resistance of winding.

Nominal diameter mm	Gauge number SWG	Overall diameter mm	Resistance at 20° C Ω/Km	Area of bare conductor mm^2
0.025	50	0.036	34026	0.0005067
0.030	49	0.041	23629	0.0007297
0.041	48	0.051	13291	0.001297
0.051	47	0.064	8507	0.002027
0.061	46	0.074	5907	0.002919
0.071	45	0.086	4340	0.003973
0.081	44	0.097	3323	0.005189
0.091	43	0.109	2626	0.006567
0.102	42	0.119	2127	0.008107
0.112	41	0.132	1758	0.009810
0.122	40	0.142	1477	0.011675
0.132	39	0.152	1258	0.013701
0.152	38	0.175	945.2	0.018242
0.173	37	0.198	735.9	0.02343
0.193	36	0.218	589.1	0.02927
0.213	35	0.241	482.2	0.03575
0.234	34	0.264	402	0.04289
0.254	33	0.287	340.3	0.05067
0.274	32	0.307	291.7	0.05910
0.295	31	0.330	252.9	0.06818
0.315	30	0.351	221.3	0.07791
0.345	29	0.384	183.97	0.09372
0.376	28	0.417	155.34	0.1110
0.417	27	0.462	126.51	0.1363
0.457	26	0.505	105.02	0.1642
0.508	25	0.561	85.07	0.2027
0.559	24	0.612	70.3	0.2452
0.610	23	0.665	59.07	0.2919
0.711	22	0.770	43.40	0.3973
0.813	21	0.874	33.23	0.5189

Nominal diameter mm	Gauge number SWG	Overall diameter mm	Resistance at 20° C Ω/Km	Area of bare conductor mm^2
0.914	20	0.978	26.26	0.6567
1.106	19	1.082	21.27	0.8107
1.219	18	1.293	14.768	1.167
1.422	17	1.01	10.85	1.589
1.626	16	1.709	8.307	2.075
1.829	15	1.920	6.654	2.627
2.032	14	2.129	5.317	3.243
2.337	13	2.441	4.020	4.289
2.642	12	2.756	3.146	5.480
2.946	11	3.068	2.529	6.818
3.251	10	3.383	2.077	8.302
3.658	9	3.800	1.640	10.51
4.064	8	40219	1.329	12.97

6 Transformer

Unlike that inductor, the transformer consists of more than one winding. Also, in order to keep the magnetisation current low, the transformer does not have airgap in its magnetising circuit.

Consider a transformer with single primary and single secondary as shown in Fig. 8. Let the

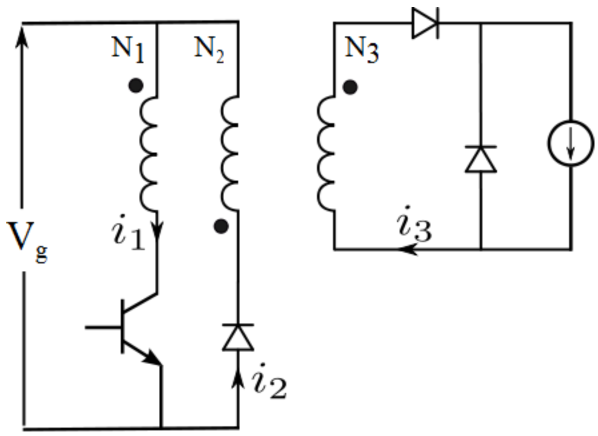


Figure 8

specifications be

$$\begin{aligned}
&\text{Primary} &&= \mathrm{V_1} \text{ volts ; } \mathrm{I_1} \text{ amps ;}\\
&\text{Secondary} &&= \mathrm{V_2} \text{ volts ; } \mathrm{I_2} \text{ amps ;}\\
&\text{VA} &&= \text{VA voltamps} = \mathrm{V_1 I_1} = \mathrm{V_2 I_2}\\
&\text{Frequency} &&= \text{f Hz .}
\end{aligned}$$

For square wave of operation, the voltage of the transformer is

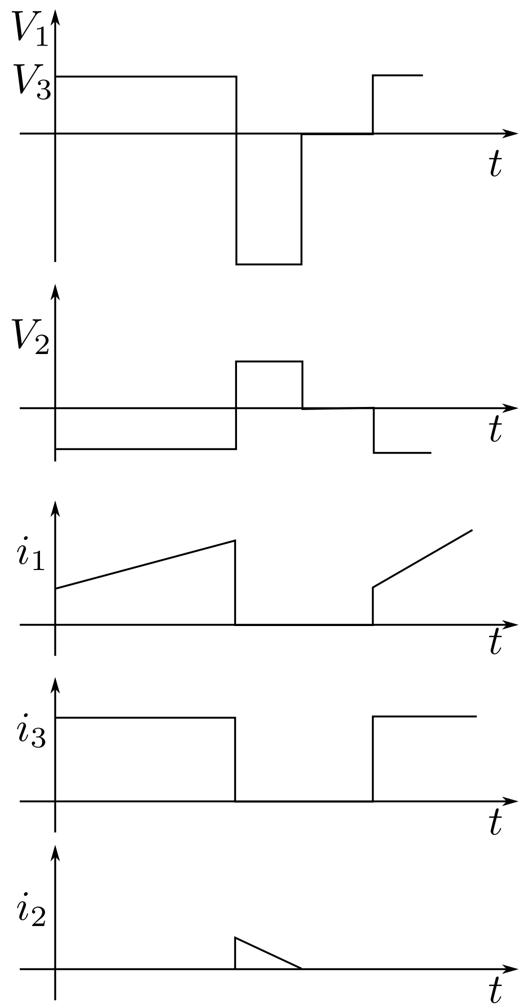


Figure 9

$$\mathrm{V_1} \;=\; \mathrm{4fB_mA_cN_1} \;;\; \mathrm{V_2} \;=\; \mathrm{4fB_mA_cN_2}$$

The window of the transformer accommodates both the primary and the secondary. With the same notation as for inductors,

$$\mathrm{k_wA_w} \;=\; \mathrm{(N_1I_1 +\; N_2I_2)/J} \;;$$
$$\mathrm{N_1I_1 = N_2I_2} \text{ (because } \mathrm{V_1I_1 = V_2I_2} \text{)}$$
$$\mathrm{N_1I_1 =\; k_wA_wJ/2;\; N_2I_2 =\; k_wA_wJ/2;}$$

From the above equations,

$$\mathrm{V_1I_1 +\; V_2I_2} \;=\; \mathrm{4k_wJB_mfA_wA_c} \;;$$
$$\mathrm{VA =\; 2k_wJB_mfA_wA_c} \;;$$

7 Transformer design

For a given specification of VA, V_1, V_2, J, B, k_w and f, it is desired to design a suitable transformer. The design requires

1. Size of the wire and number of turns to be used for primary and secondary windings.

2. Core to be used.
3. Resistance of the winding.
4. Magnetising inductance of the transformer.

8 Design steps

1. Compute the Area product of the desired core.
   $A_c A_w = VA/(2fk_w J B_m)$
2. Select the smallest core from the core tables having an area product higher than obtained in step (1).
3. Find the core area (A_c) and window area (A_w) of the selected core.
4. Compute the number of turns.
   $N_1 = V_1/4fB_m A_c$; $N_2 = V_2/4fB_m A_c$
5. Select the nearest higher whole number to that obtained in step (4), for the primary & secondary turns.
6. Compute the wire size required for secondary & primary.
   $a_{w1} = I_1/J$; $a_{w2} = I_2/J$
7. Select from the wire tables the desired wire size.
8. Compute the length of secondary & primary turns, from the mean length per turn of the core tables.
9. Find from the wire tables , the primary & secondary resistance.
10. Compute from the core details, the reluctance (R) of the core.
11. Compute the magnetising inductance.
    $L_m = N^2/R$

9 Problems

Design the following inductors and transformers. Use the wire and core tables given in the class. Make suitable assumptions. Assume suitable values for J, B_m, k_w, etc.

1. An inductor of 2 mH, capable of carrying a current of 3 A dc.
2. An inductor of 2 mH, capable of carrying a current of 3 A ac, at 50 Hz.
3. A 50 VA transformer operating at 15 KHz, square wave with primary and secondary voltages of 20 & 40 V respectively.
4. Fig. 8 shows a forward converter. It employs a transformer with with three windings. Required data and the voltage and current waveforms of the transformer windings are shown. Make a design of this transformer.

GUEST KEEN WILLIAMS LIMITED
SANKEY PRESSINGS DIVISION

GENERAL PROPERTIES

COLD ROLLED GRAIN ORIENTED ELECTRICAL STEEL

Grade	Thickness mm	Approx. Silicon %	Density gms/cm^3	Insulation AISI Type	Minimum Stacking Factor %	Guaranteed Iron Loss at 50 Hz - Watts/kg.	
						1.5T	1.7T
M4	0.27	3.2	7.65	C5	94	0.89	-
M5	0.30	3.2	7.65	C5	94	0.97	-
M6	0.35	3.2	7.65	C5	95	1.11	-
M2H	0.30	3.2	7.65	C5	94	-	1.17
M3H	0.30	3.2	7.65	C5	94	-	1.23

Note: The figures given above represent typical values. They may be used as a guide by designers but no guarantee is implied.

Insulation :

(i) Composition : Inorganic

(ii) Interlaminar Resistance : Higher Interlaminar Resistance.

(iii) Rust Resistant : Transformer Oil, Refrigerants and Lubricants have no effect on the coating.

Type No.	Ref. Fig.	Nominal Dimensions (cm) A	B	C	D	E	F	G	Rivet No.	Holes Dia. (cm)	Weight per 100 pairs (Kg) HR B710	CRNO S23	CRNO S10	CRGO M6	Mean M... Length ...
6	1	3.81	12.70	11.43	1.91	1.91	7.62	2.54	4	0.635	4.08	4.08	2.82	2.82	27.9
INT-120	1	4.00	12.00	10.00	2.00	2.00	6.00	2.00	4	0.600	-	-	-	2.54	24.0
43	1	5.08	15.24	12.70	2.54	2.54	7.62	2.54	4	0.675	5.94	5.94	4.11	4.11	30.48
8	1	5.08	18.45	17.15	2.54	2.54	12.07	4.13	4	0.952	8.26	8.26	5.71	5.71	42.55
INT-180	1	6.00	18.00	15.00	3.00	3.00	9.00	3.00	4	0.950	8.26	8.26	-	5.71	36.00
8A	1	7.62	25.40	21.59	3.81	3.81	13.97	5.08	4	0.950	15.64	15.64	-	-	53.3
8B	1	7.62	23.50	19.69	3.81	3.81	12.07	4.13	4	1.100	13.91	13.91	9.61	9.61	47.6
8C	1	7.62	23.50	31.75	3.81	3.81	24.13	4.13	4	1.100	21.04	-	-	14.54	71.7
100	1	10.16	35.56	25.40	5.08	5.08	15.24	7.62	6	0.952	25.83	25.83	17.85	17.85	66.0
4AX	2	2.38	9.05	8.10	1.11	1.11	5.87	4.45	4	0.397	-	-	1.25	-	25.16
35A	2	3.81	15.88	13.34	1.91	1.91	9.53	8.26	4	0.635	5.11	-	-	-	40.8
32	3	0.64	2.54	1.27	0.32	0.32	1.91	0.64	-	-	-	-	0.09	0.09	6.35
28	4	1.27	3.81	5.40	4.13	-	6.99	1.27	-	-	0.42	-	-	-	2.16
43 (T.P.)	1	2.54	15.24	12.70	2.54	2.54	7.62	3.81	4	0.675	5.19	5.19	-	3.59	31.75
8B (T.P.)	1	3.81	23.50	19.69	3.81	3.81	12.07	6.03	4	1.100	12.10	12.10	-	8.38	49.53
100 (T.P.)	1	5.08	35.56	25.40	5.08	5.08	15.24	10.16	6	0.952	22.80	22.80	15.80	15.80	68.58

(i) Weights given are only indicative and not guaranteed.
(ii) Laminations are manufactured only from the electrical steel grade under which they are listed.
(iii) T.P. denotes Three Phase.
(iv) Lamination type nos. 3,5,15,16,33 and 43 are available with air gap on centre limb for choke application. Details on request.

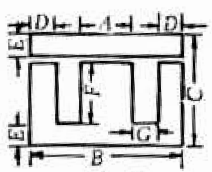


Fig. 1
E AND I TYPE

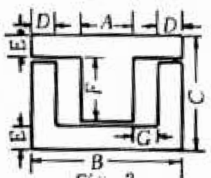


Fig. 2
T AND U TYPE

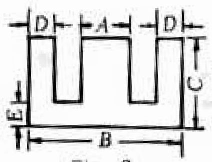


Fig. 3
E TYPE

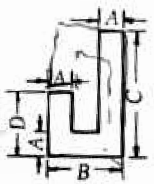


Fig.4
J TYPE

SPECIFICATIONS OF ELECTRICAL STEELS (INSULATED ON BOTH SIDES

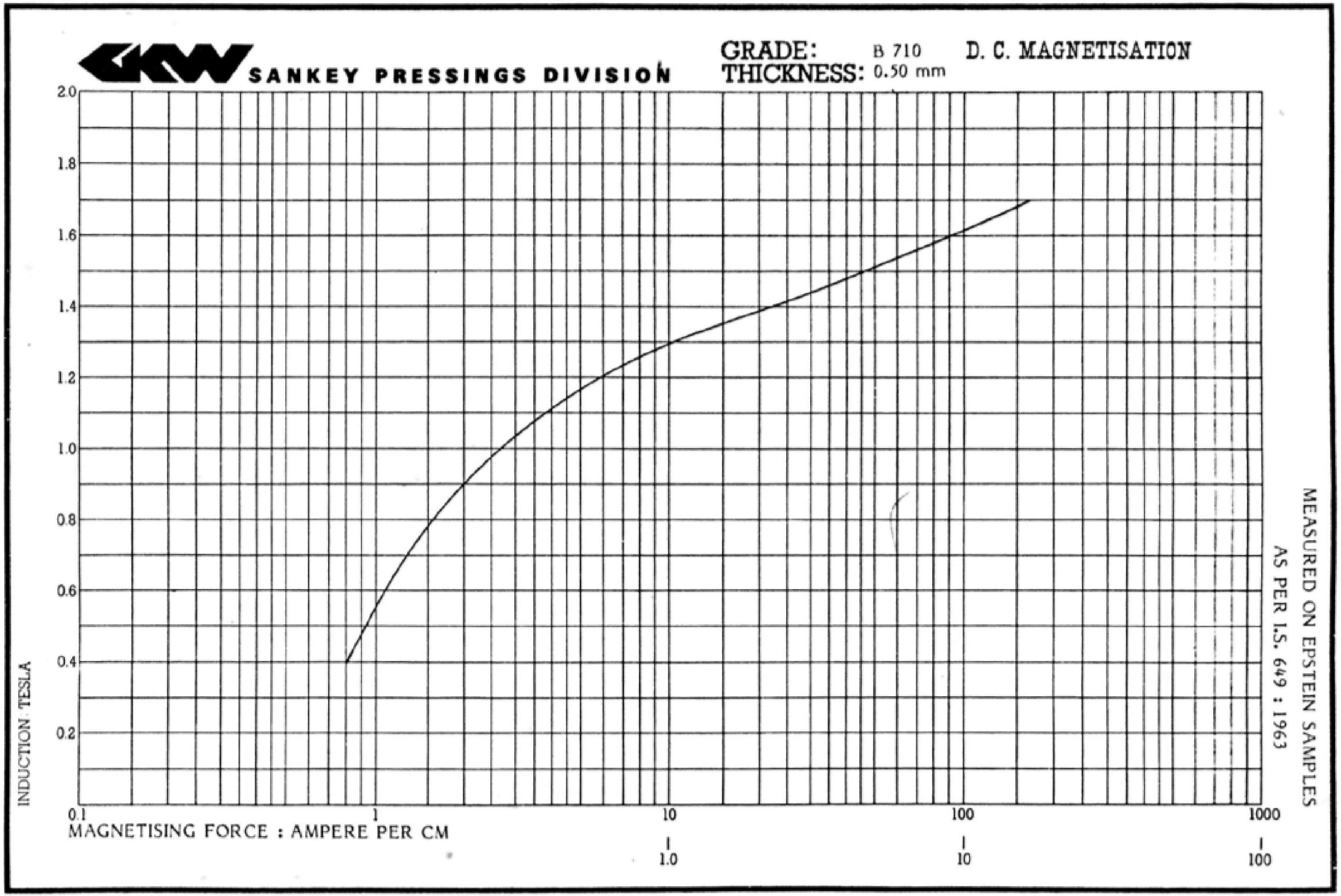
Grade of Electrical Steel	Nominal Thickness mm	Density gms/cc	Guaranteed Iron Loss at 1.5T, 50Hz-W/kg.	Stack... Fact %
HR B710	0.50	7.75	7.10	94
CRNO S23	0.50	7.75	6.20	96
CRNO S10	0.35	7.65	2.65	94
CRGO M6	0.35	7.65	1.11	95

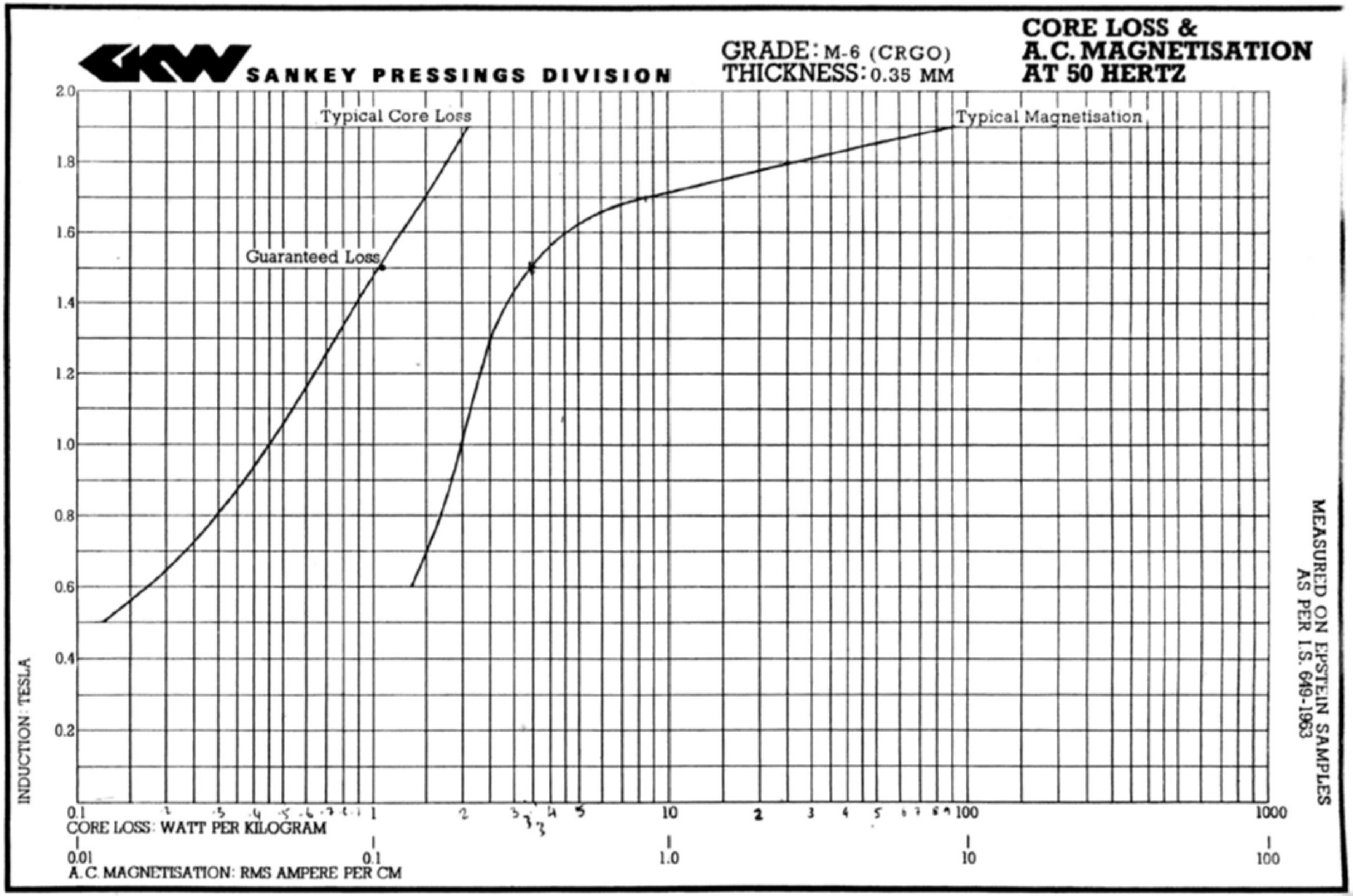
GUEST KEEN WILLIAMS LIMITED

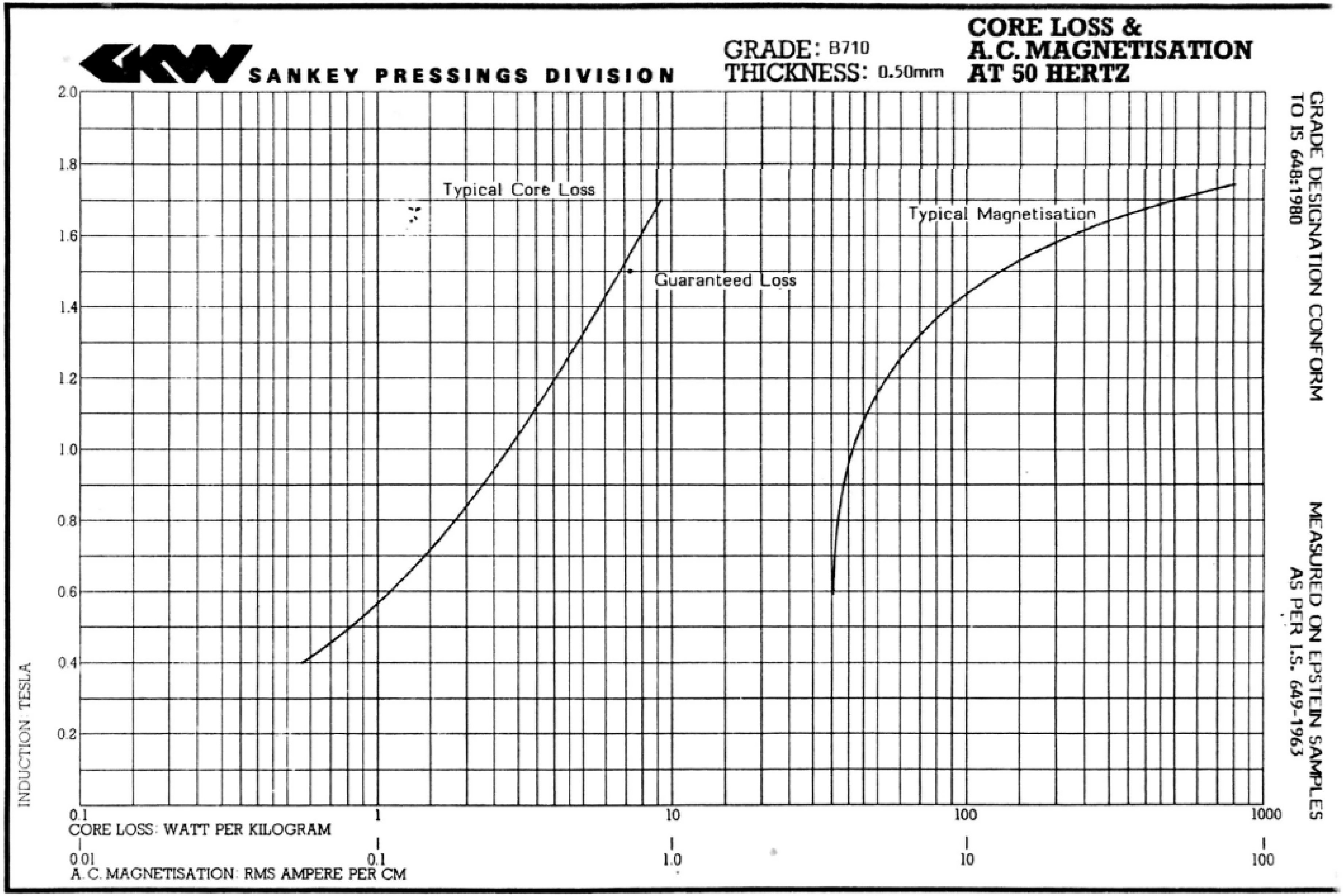
SANKEY ELECTRICAL STAMPINGS DIVISION
Lal Bahadur Shashtri Marg, Bhandup, Bombay 400078.

Transformers and Choke Laminations
STANDARD TYPES

Type No.	Ref. Fig.	Nominal Dimensions (cm)							Rivet Holes		Weight per 100 pairs (Kg)				Mean Mag. Length (cm)
		A	B	C	D	E	F	G	No.	Dia. (cm)	HR B710	CRNO S23	CRNO S10	CRGO M6	
L 202	1	0.35	1.40	1.14	0.18	0.18	0.79	0.35	-	-	-	-	-	0.028	2.98
L 164	1	0.48	1.91	1.59	0.24	0.24	1.11	0.48	-	-	-	-	-	0.053	4.13
L 109	1	0.64	2.54	1.91	0.32	0.32	1.27	0.64	-	-	-	-	-	0.086	5.08
12 AX	1	0.95	4.13	3.49	0.64	0.64	2.22	0.95	-	-	-	-	-	0.273	8.73
17	1	1.27	3.81	3.18	0.64	0.64	1.91	0.64	-	-	0.375	0.375	0.259	0.259	7.62
INT-41	1	1.30	4.10	3.30	0.60	0.60	2.10	0.80	-	-	-	0.394	-	-	8.25
17 A	1	1.43	4.29	3.57	0.71	0.71	2.14	0.71	-	-	-	0.476	-	0.329	8.57
12 A	1	1.59	4.76	3.97	0.79	0.79	2.38	0.79	-	-	0.586	0.586	0.405	0.405	9.53
10 A	1	1.59	6.03	5.40	0.95	0.95	3.49	1.27	4	0.318	-	-	0.625	-	13.17
1	1	1.67	6.43	5.72	0.79	0.79	4.13	1.59	-	-	0.916	0.916	0.633	-	14.65
74	1	1.75	5.24	4.37	0.87	0.87	2.62	0.87	-	-	0.709	0.709	0.490	0.490	10.48
23	1	1.91	5.72	4.76	0.95	0.95	2.86	0.95	-	-	0.844	0.844	0.583	0.583	11.42
2	1	1.91	7.62	7.62	0.95	0.95	5.72	1.91	-	-	1.406	1.406	0.972	-	18.10
30	1	2.00	6.00	5.00	1.00	1.00	3.00	1.00	4	0.397	0.911	0.911	0.630	0.630	12.00
45	1	2.22	6.67	5.56	1.11	1.11	3.33	1.11	4	0.396	-	-	0.780	-	13.33
31	1	2.22	6.67	5.56	1.11	1.11	3.33	1.11	-	-	-	1.151	0.795	-	13.33
15	1	2.54	7.62	6.35	1.27	1.27	3.81	1.27	4	0.556	1.462	1.462	1.010	1.010	15.24
14	1	2.54	8.41	6.67	1.35	1.27	4.13	1.59	4	0.556	1.628	-	1.125	-	16.59
33	1	2.80	8.40	7.00	1.40	1.40	4.20	1.40	4	0.437	1.800	1.800	1.244	1.244	16.80
3	1	3.18	9.53	7.94	1.59	1.59	4.76	1.59	4	0.635	2.29	2.29	1.59	1.59	19.06
16	1	3.81	11.43	9.53	1.91	1.91	5.72	1.91	4	0.556	3.34	3.34	2.31	2.31	22.86
5	1	3.81	12.07	9.53	1.91	1.91	5.72	2.22	4	0.675	3.41	3.41	2.36	-	23.49


SANKEY PRESSINGS DIVISION
GRADE: B 710
THICKNESS: 0.50 mm
D. C. MAGNETISATION
INDUCTION: TESLA
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.1
1
10
100
1000
1.0
10
100
MAGNETISING FORCE : AMPERE PER CM
MEASURED ON EPSTEIN SAMPLES
AS PER I.S. 649 : 1963


GKW SANKEY PRESSINGS DIVISION
GRADE: M-6 (CRGO)
THICKNESS: 0.35 MM
CORE LOSS & A.C. MAGNETISATION AT 50 HERTZ
Typical Core Loss
Typical Magnetisation
Guaranteed Loss
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
INDUCTION: TESLA
0.1
1
10
100
1000
CORE LOSS: WATT PER KILOGRAM
0.01
0.1
1.0
10
100
A.C. MAGNETISATION: RMS AMPERE PER CM
MEASURED ON EPSTEIN SAMPLES AS PER I.S. 649-1963


SANKEY PRESSINGS DIVISION
GRADE: B710
THICKNESS: 0.50mm
CORE LOSS &
A.C. MAGNETISATION
AT 50 HERTZ
Typical Core Loss
Guaranteed Loss
Typical Magnetisation
INDUCTION TESLA
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.1
1
10
100
1000
CORE LOSS: WATT PER KILOGRAM
0.01
0.1
1.0
10
100
A.C. MAGNETISATION: RMS AMPERE PER CM
GRADE DESIGNATION CONFORM
TO IS 648:1980
MEASURED ON EPSTEIN SAMPLES
AS PER I.S. 649-1963